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The Greek Modes: Working Our Way Out of the Enharmonic Cave

Abstract

Only two incorrectly described scales of Aristides Quintilianus prevent us from a better understanding of the Greek modes. This relatively small historical error has mushroomed into what I call the Enharmonic Cave in which we now live. Our task is to work our way out of this dilemma by a fresh analysis of the “Damonian” scales of Aristides. My Two-Source Hypothesis will supply the justification for correcting the two faulty scales, Lydian and Tense Lydian. I will conclude with how we have exacerbated the problem of the Enharmonic Cave by continuing to accept the highly questionable Westphal Emendation of 1883, along with Andrew Barker’s problematic “empiricist” interpretation, both of which enslave us further in the Cave, and offer no way out.

Keywords: *ancient Greek modes; Aristides Quintilianus; enharmonic genus; octave species; harmonic theory.*

The Common Practice Period of Western music history (17th to 19th centuries CE) hinders our understanding of modal music. I call this period the Bubble, because despite the wide modal variety in our Greek documents (Pöhlmann-West 2001) in Gregorian chant, in the Renaissance, and in today’s music, the Bubble is a modal wasteland. Only one pure mode, the C-mode, fits into the “tonality” Bubble. The A-mode only fits because we force it to look like a C-mode, by adding the leading tone

(semitone below the tonic). The leading tone's dominance originates in 14th-century vocal polyphony, but in Gregorian chant and the Greek documents, the leading tone is rare. In the Bubble, the tritone is always dissonant, but there are plenty of tritones in the Greek documents. So when R. P. Winnington-Ingram (1936), Martin West (1992a) or Andrew Barker (1984 and 1989) say "leading tone," or think the tritone is dissonant, they are trapped in the Bubble.

We also must understand the harmonic series because its low end contains important musical intervals. Assuming that the harmonic series is based on the note *c*, the first eight harmonics, in ascending pitch from left to right, are: *c-c-g-c-e-g-b^b-c*.¹ In the first two octaves (*c-c*, *c-g-c*), the Greeks recognized the "consonances" of the octave (*c-c*), the fifth (*c-g*), and the fourth (*g-c*). The third octave (*c-e-g-b^b-c*) has what I call two "character" notes: *e* and *b^b*. The note *e* creates the major third, *c-e*, and the note *b^b* creates the septimal third, *g-b^b*. We say "septimal" because *b^b* is the seventh harmonic. These two thirds will determine two of the four modal characters.

There are many aspects to the Greek modes: scales, rhythms, melodic formulas, favored tessituras, tempos, dynamics, etc. I do not deny any of these aspects. I only focus on the scales, first by finding tonics. A tonic is simply a note of orientation, a focal note around which other notes are oriented. It does not have to occur first, or last, or the most times in a melody, although it often does in these three cases. It is a note that we are somehow able to keep in our musical memories. Let us compare C-major, *c-d-e-f-g-a-b-c*, with the most common form of minor in the Bubble, C-harmonic minor, *c-d-e^b-f-g-a^b-b-c*. Excluding the

¹ Musical notes are in italics, and the choice of *c* is arbitrary. The harmonic series arises from natural acoustics.

tonic (*c*), four notes are common (*d, f, g, b*). These scales only differ by two notes (*e/e^b, a/a^b*). So, major and minor depend on a small subset of notes, not all of the notes. In fact, we say this scale, *c-d-e-f-g-a^b-b-c*, is C-major with a borrowed *ab* from C-minor. So, it is the *e/e^b* which determines whether we say C-major or C-minor. In summary, one note is tonic (*c*), there are one or two character notes (*e/e^b, a/a^b*), and four notes are not as significant in determining mode (*d, f, g, b*). This is the strategy that we want to use when analyzing Greek scales: look for a tonic, then look for one or two character notes.

Aristides' Damonian Scales

	*F	A	B*C	E*	Lydian	} Joy
	E*F	A	C	D	Iastian	
D	E*F	A	B*C	E	Dorian	Courage
D	E*F	A	B*C	D	Phrygian	Frenzy
	E*F	G	A*B^b	E	Mixolydian	} Sorrow
	E*F	A	C		Tense Lydian	

fig. 1 – Aristides' Damonian Scales

Figure 1 shows the Greek scales to analyze (Barker 1989, 419-420, and 1984, 130-131, 175-176, 181; West 1992a, 174). Welcome to the Enharmonic Cave, because every scale is enharmonic. In his three-book treatise, *On Music*, Aristides Quintilianus introduces these “Damonian” scales (West’s term) in Book One, chapter 9, which he then relates to Plato’s *Republic*. The modal names are the same as those in the *Republic*. On the far right side are the four modal characters.² The stars (*), located a quartertone from their neighbors (*e, f, b, c, a, b^b*), are notes of less importance because they can be

² The frenzy of Phrygian comes from Aristotle.

omitted without affecting the enharmonic character of the scales.³ The notes that are in boxes are tonics that take advantage of the acoustical resonance of the octave. To achieve this, the octave scales that are divided fourth below fifth should have a tonic in the middle: Dorian and Mixolydian (*e-a-e*, tonic is on *a*). And those octave scales that are divided fifth below fourth should have a tonic on the bottom: Lydian (*-*-**e*, tonic is on low star), Dorian (*e-b-e*, tonic is on low *e*), and Phrygian (*d-a-d*, tonic is on low *d*). Notice that Dorian has two tonics: *a* and low *e*.

Analysis of Aristides' Damonian Scales

When I began analyzing these scales back in 2012, I had to start with what I knew — major and minor triads. Especially major, because we know the Greeks responded to the first three intervals of the harmonic series (*c-c-g-c-e-g^b-c*), i.e. the octave (*c-c*), the fifth (*c-g*), and the fourth (*g-c*), and I suspected that they also responded to the very next interval, the major third (*c-e*), even though they did not consider it a consonance. Let us start with our tonics in the boxes above: Two scales have minor triads on those tonics (Dorian, **a**-*c-e*, Phrygian, **d**-*f-a*, tonics are in bold and underlined). But no scales have major triads on those tonics. In the Lydian scale, **f-a-b*c-e*, the note *a* is too high. It is a major third plus a quartertone above the tonic (the low star). So let us check for a major triad in the last two (non-octave) scales, Iastian, *e*f-a-c-d*, and Tense Lydian, *e*f-a-c*. Since both have the major triad, *f-a-c*, let us check the character of each mode: joy (Iastian), and sorrow (Tense Lydian). My intuition linked the major triad with Iastian's joy. So the tonic is

³ Barker, 1984, 215-217; West 1992a, 163-164, 176. The Lydian scale is an exception (see below).

on **f** in the Iastian scale, $e^*f-a-c-d$, whose character is governed by the major triad, **f-a-c**.⁴

Looking at the harmonic series again, $c-c-g-c-e-g-b^b-c$, the major triad, $c-e-g$, represents harmonics 4-5-6. The septimal third, $g-b^b$, represents harmonics 6-7. Now, Bubble people⁵ have difficulty recognizing the stable septimal third, because the typical 7th-chord in the Bubble is unstable and must resolve. We have to go outside the Bubble to recognize the stable septimal third. It is found in Medieval chant (e.g. *Gloria IX (Cum júbilo)*, *Kyrie VI (Kyrie Rex Genitor)*) and in rock music (e.g. Led Zeppelin's *Whole Lotta Love* (base line)). Of Aristides' scales, only Phrygian, $\underline{d}-e^*f-a-b^*c-d$, has a septimal third: $a-b^*c$ (with b and $*$ as "infix" notes). Considering $a-b-c$ without the optional star ($*$), notice that changing b to bb would alter the sound of the septimal third: Sing up and down the tune, $\underline{d}-a-b-c-b-a-\underline{d}$, then sing it again, changing b to b^b : $\underline{d}-a-b^b-c-b^b-a-\underline{d}$. The b emphasizes the c , the seventh harmonic, while the b^b emphasizes the a . This is a subtle yet important distinction. So, *the tonic is on low d in the Phrygian scale, $\underline{d}-e^*f-a-b^*c-d$, whose character depends on the septimal third with its infix note b : $a-b^*c$ (the star is optional)*. The first two modes of our analysis so far, Iastian and Phrygian, come from the same source: the third octave of the harmonic series.

Here is where I got stuck in the Enharmonic Cave for one year (2012-2013), unable to make any progress. What bothered me was the Lydian scale, $f-a-b^*c-e$, because to match Iastian's joyful character, I needed a major triad built on the low star ($*$)

⁴ I prioritize a over c , because omitting c still preserves the major third, **f-a**, but omitting a produces an open fifth, **f-c**, which can be major (**f-a-c**) or minor (**f-a^b-c**).

⁵ I also include myself. The Bubble's influence cannot be overstated.

as tonic, but the note *a* was too high by a quartertone. Andrew Barker⁶ suggested putting the tonic on *f*, giving me the same major triad as Iastian (*f-a-c*), and doubling the leading tone, i.e. the low star, duplicated an octave higher (high star). But doubling the leading tone is Bubble thinking. The leading tone rarely occurs in our Greek documents, so I certainly will not double it. The major triad must have its tonic at the bottom of the scale, but the only way I could make the note *a* go down by a quartertone was to make the whole scale chromatic: *f-f[#]-a-b-c-c[#]-e-f*. So I deliberately broke the enharmonic rule. I now assumed that Aristides' Lydian scale should be chromatic, and I ignored the enharmonic Lydian scale since it gave me problems. Thus, *the tonic is on low f in the new Lydian scale of Aristides, whose genus should be chromatic: f-f[#]-a-b-c-c[#]-e-f, and whose character depends on the major triad, f-a-c, just like Iastian.*

Plato suggests that Iastian and Lydian are “slack.” Notice that in Iastian (*e*f-a-c-d*) and in our new chromatic Lydian (*f-f[#]-a-b-c-c[#]-e-f*), the tonic *f* occurs on different movable notes of the tetrachord: *lichanos* in Iastian, *e*f-a*, and *parhypate* in Lydian, *[e]-f-f[#]-a*. Yet the tonic for both is *f*. Now consider Tense Lydian, *e*f-a-c*. If its tonic is on the same note, *f*, but we apply “tension” to it (like a string), then the tonic will rise in pitch up to *f[#]*, giving us a new chromatic Tense Lydian scale: *e-f-f[#]-a-c*, with a new characteristic sound, the diminished triad, *f[#]-a-c*. Sing up and down the major triad, *f-a-c* (joy), and the diminished triad, *f[#]-a-c* (sorrow), and notice the vast change in character. The “tense” in Tense Lydian means “raise the tonic” from *f* to *f[#]*, so that the tonic is on *f[#]* in the new Tense Lydian scale of Aristides, whose genus should be chromatic: *e-f-f[#]-a-c*, and whose character depends on the diminished triad, *f[#]-a-c*.

⁶ Andrew Barker, personal communication (May 2020).

The Greeks do not have a word for tonic. They have code words: “slack” means tonic on f , and “tense” means tonic on $\underline{f}^\#$. Our new chromatic scales, Lydian and Tense Lydian (figure 2), situate us outside the Enharmonic Cave. They should replace their enharmonic counterparts in figure 1.

Chromatic Scales: Lydian and Tense Lydian



fig. 2 – Chromatic Scales: Lydian and Tense Lydian

Let us look for a diminished triad in Mixolydian, $e^*f-g-\underline{a}^*b^b-e$, as it has the same woeful character as Tense Lydian. A diminished triad on tonic $\underline{a}$ would be $\underline{a}-c-e^b$, which does not fit. But there is a diminished triad on the low e , which should be our new tonic: $\underline{e}-g-b^b$. Notice that our new tonic is doubled at the top (high e). So, we initially chose the wrong tonic, a . Instead, *the tonic is on low $\underline{e}$ in the Mixolydian scale, $\underline{e}^*f-g-a^*b^b-e$, whose character depends on the diminished triad, $\underline{e}-g-b^b$* . Its name comes from a mixture of Dorian and the diminished triad of Tense Lydian, now based on low e . Both Mixolydian and Tense Lydian are connected with high pitch (Barker 1984, 166). So singing diminished triads at a higher pitch would likely intensify the sorrowful character of these modes. Lines 16-19 of Berlin Papyrus 6870 are a lament for the suicide of Ajax, in which a B-minor triad ($\underline{b}-d-f^\#$) and a B-diminished triad ($\underline{b}-d-f$) are used. The B-diminished triad is in line 17 (at the end), and in line 19. Both triads are combined at the end of line 18. Please notice that in the score, the note f should replace the note $\uparrow e$.⁷

⁷ Pöhlmann-West 2001 56-60. The editors assume that the note f must be a quartertone lower in pitch ($\uparrow e$) because it is notated with an epsilon, not a

Are there any modern analogies to the modal characters? I think most people can link the major triad with joy. The septimal third with its infix (a-b-c) is the basis of the “Phrygian” tetrachord (a-b-c-d),⁸ which I associate with rock’s frenzy. Rock highlights the Phrygian tetrachord in many ways. One way is the chord progression, $d^7-G^7-d^7-G^7-d^7-G^7\dots$ without ever resolving to C, because “resolving to C” is Bubble thinking. This is a Phrygian progression in the d-tonic Phrygian scale, d-e-f-g-a-b-c-d, and the Phrygian tetrachord (a-b-c-d) governs the chord progression: a, c, and d are in the d^7 tonic chord, b and d are in G^7 . An example is Stevie Wonder’s *I Wish*. As for relating the diminished triad with sorrow, the only genre that I can think of is the blues, because the defining note of the blues scale is the “diminished fifth note” placed in a minor pentatonic scale, e.g. the g^b in the C-blues scale: c-e^b-f-g^b-g-b^b-c. Sing it up and down. The descending motive, g^b -f-e^b-c, outlines a diminished triad (c-e^b-g^b), but I will not push this analogy too far because g^b can also be heard as a neighboring tone.

Before analyzing Dorian, there are two other modes that are not in Aristides’ Damonian scales. We know them as octave species. One is called Aeolian, identified as the Hypodorian octave species: a-b*c-e*f-a. It is enharmonic because

delta, representing the fine tuning of Aristoxenian theory. Assuming that notators always notate according to theory is Bubble thinking. Sometimes notators use a much simpler, easier way to notate, by choosing the “next symbol,” regardless of key. In this case, zeta is the note e and the notator chose the next letter, epsilon = f. Other documents in the same collection of sources also use this method: document number 5 (nu is b), 6 (nu is b, delta is e), 18 (upsilon is g), 24 and 25 (nu should be b, not c[#]), 39 (lambda is c).

⁸ Winnington-Ingram 1936, 27. The Phrygian tetrachord = [whole tone, semitone, whole tone].

Hypodorian means “similar to Dorian,” and the enharmonic genus is very well suited to Dorian.⁹ The scale of Aeolian (Hypodorian) is divided fifth below fourth (*a-e-a*), so a tonic on the bottom would take advantage of the octave’s acoustical resonance. Therefore, the tonic is on low a in the Aeolian mode, identified as the Hypodorian octave species: a-*b***c*-*e***f*-*a*.

Aeolian is said to be “between” relaxed and tense Ionian (= Iastian).¹⁰ We already have a “relaxed” (slack) Iastian in Aristides, but we do not have a tense Iastian. However, since I associated “tense” with the diminished triad, I will substitute Aristides’ Tense Lydian for tense Iastian. I then interpret the minor triad of Aeolian, a-*c*-*e*, as being between the major triad of relaxed Iastian, f-*a*-*c*, and the diminished triad of Tense Lydian, f[#]-*a*-*c*. To hear this, sing all three triads transposed to the tonic of a. Sing up and down: first major, a-*c*[#]-*e*, then minor, a-*c*-*e*, and then diminished, a-*c*-*e*^b.

The best way to fit Aeolian into the Damonian scales is to put the tonic on low d and use the conjunct tetrachord (*a***b*^b-*d*), resulting in d-*e***f*-*a***b*^b-*d*. Notice that Aeolian has *b*^b, but Phrygian does not: d-*e***f*-*a*-*b***c*-*d*. It is this “minor sixth” note (*b*^b), a semitone above the fifth scale degree (*a*), that characterizes Aeolian. Otherwise, Aeolian and Phrygian have the same lower fifth: d-*e***f*-*a*. The low d is called *hyperhypate*, which is another code word for tonic, this time for Aeolian and Phrygian, since early Dorian tunes did not use *hyperhypate* (Barker 1984, 224).

The other mode that is not in the Damonian scales, and is only known as an octave species, is Hypophrygian. It should be “similar to Phrygian” so I will look for the septimal third.

⁹ Barker 1984 282; West 1992a, 175, n. 49, 183-184, 342.

¹⁰ West 1992a, 178, 342.

Aristides hints that Hypophrygian is diatonic: *g-a-b-c-d-e-f-g* (Barker 1989, 417, with note 100; cf. West 1992a, 186, n. 100). Thus, its septimal third, with infix, is *d-e-f*. Actually, Hypophrygian must be diatonic because its mese must be a fourth below Phrygian's mese (see figure 4). So, *the tonic is on low **g** in the diatonic Hypophrygian octave species, **g**-a-b-c-d-e-f-g, whose character (frenzy, same as Phrygian) depends on the septimal third with its infix note e: d-e-f.*

There are nuances to Hypophrygian: Focusing on the major triad, **g-b-d**, at the expense of the septimal third with its infix, *d-e-f*, might cause Hypophrygian to sound more like Iastian. Or having the septimal third below the tonic, as in the G-mode, *d-e-f-g-a-b-c-d*. I suggest that the G-mode, **g-a-b-c-d-e-f-g**, is more effective due to harmonics 4-5-6-7-8 (in bold) of the harmonic series based on g: *g-g-d-g-b-d-f-g*. Hypophrygian fits into the Damonian scales best by putting the tonic on low **d**, resulting in **d-e-f[#]-g-a-b-c-d**. When we compare it with the diatonic Phrygian scale, **d-e-f-g-a-b-c-d**, they both have the same upper fourth, *a-b-c-d* (Phrygian tetrachord), but they have different triads in the lower fifth: **d-f[#]-a** and **d-f-a**.

What about Dorian: *d-**e***f-**a**-b*c-e*? Dorian was the most difficult to analyze because my method of finding one or two character notes did not work. One of the reasons is conjunct modulation, yielding *d-**e***f-**a***b^b-d*. The tonic on low **e** now produces a B-mode, because of the “flat fifth” (*b^b*). Also some brief passages in the second Delphic hymn suggest a tonic on paramese (a B-mode).¹¹ The old Hypodorian *tonoi* of the *tonoi* systems before Aristoxenus may suggest a B-mode.¹² As a

¹¹ Pöhlmann-West 2001, 75 (lines 2, 4, 5), 83 (line 34).

¹² Barker 1989, 153-154. Regrettably, a discussion of the *tonoi* is beyond the scope of this paper.

result, there are *three* tonic locations, e, a, and b, for Dorian: d-e*f-a-b*c-e, and there are no character notes. The best way to get Dorian in our ears is to sing each tonic with its extended neighbors. Tonic on a will sound familiar, tonic on e will be less familiar, and tonic on b will sound rather alien. Sing b's neighbors: b-c-b-a-f-a-b. Then sing a's neighbors: a-b-c-b-a-f-a. Finally, sing low e's neighbors: e-f-a-f-e-d-e.

The tonic is moved up to the top of the Dorian scale (nete) in the Delphic hymns:¹³ first hymn, lines 17-26 (tonic on a); second hymn, lines 8-14, 18-23, 26-30 (tonic on e), lines 31-33 (tonic on a). I call this the Dorian High Melodic Style. Also in the hymns is the leap down of a tritone, characteristic of Dorian.¹⁴ Therefore the tritone, i.e. the Bubble Boogeyman,¹⁵ should be our friend. Sing it in the shower and at bedtime.

Figure 3 has the results of our analysis. These are Aristides' Damonian scales of figure 1, plus the two additional scales, Aeolian and Hypophrygian. Two of the Damonian scales, Lydian and Tense Lydian, are changed to chromatic (in green). Tonics are in boxes, character notes are circled, and characteristic intervals have arches.

¹³ Pöhlmann-West 2001, 62-85.

¹⁴ Barker 1984, 216. The tritone leap is from *paramese* down to diatonic *parhypate*.

¹⁵ Not the dancer.

Analysis of the Greek Modes

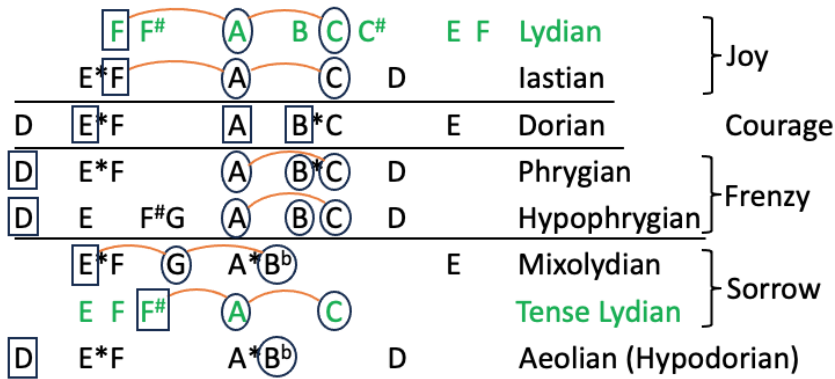


fig. 3 – Analysis of the Greek Modes

Notice that a chromatic middle tetrachord (*e-f-f[#]-a*) would include the Phrygian *f* and the Hypophrygian *f[#]*. This is the New Music: Tonic on *hyperhypate*, low *d*, with a chromatic middle tetrachord having the variable thirds, *f* and *f[#]*, together with an optional minor sixth note (*b^b*) from Aeolian (Hypodorian). This is because the chromatic genus was used in the New Music, and a source of the New Music was the dithyramb, which used Phrygian (*f*), Hypophrygian (*f[#]*), and Hypodorian (*b^b*). The modernist Agathon introduced some of these New Music elements into tragedy.¹⁶

An example of the New Music is lines 10-16 (*DAGM*, p. 65) of the first Delphic hymn, where the note *b* should be *b^b*, *hyperhypate* of the Hyperphrygian key. In the notation, the vocal sign omicron is *b^b* for the same reason that the instrumental sign nu is *f[#]* in the second Delphic hymn: Both are basic signs that move away from their neighbor by a semitone (nu up to *f[#]*,

¹⁶ Hypophrygian (*f[#]*), Hypodorian (*b^b*), and possibly the chromatic genus: West, *AGM*, 349-50, 352, 354, 369. The *f[#]* of Hypophrygian may already allude to the chromatic genus: *e-f-f[#]-a*.

omicron down to b^b).¹⁷ Sing lines 10-16 with a tonic on bb instead of b in order to hear the variable thirds, d^b and d . Sing line 15 of the second Delphic hymn (DAGM, p.79, tonic on low $\underline{g}$, *hyperhypate* of the Lydian key) in order to hear the minor sixth note, e^b .¹⁸

Lastly, here are the diatonic octave species, with tonics in bold and underlined: Mixolydian ($\underline{b}$ - c - d - e - f - g - a - b), Lydian (c - d - e - $\underline{f}$ - g - a - b - c), Phrygian ($\underline{d}$ - e - f - g - a - b - c - d), Dorian ($\underline{e}$ - f - g - a - b - c - d - e), Hypolydian ($\underline{f}$ - g - a - b - c - d - e - f), Hypophrygian ($\underline{g}$ - a - b - c - d - e - f - g), and Hypodorian ($\underline{a}$ - b - c - d - e - f - g - a). Almost all have tonics on the bottom and doubled at the octave: thus, the appeal of the octave species as a theoretical tool for governing the *tonoi*. The octave species, the *tonoi*, and the Perfect System are conceived of together, as one historical development, probably sometime in the mid-fourth century BCE.¹⁹

With Aristides' scales in figure 1, we were in the Enharmonic Cave. With the two green chromatic scales, Lydian and Tense Lydian, we are outside in the sunshine, Cave-free. This happened to me in 2013, and I spent the next several years trying to determine why Lydian and Tense Lydian are

¹⁷ Pöhlmann-West 2001, 84: last sentence, $N=f^\#$. The $f^\#$ matches the a in the first hymn, line 25: I call it the "space" motive, since the tonic's upper neighbor moves up by a semitone, giving the tonic more space. First hymn, lines 17-26, tonic on g : in line 25, the upper neighbor (a^b) moves up by a semitone (a). Second hymn, lines 18-23, tonic on $\underline{e}$: in line 23, the upper neighbor (f) moves up by a semitone ($f^\#$).

¹⁸ The New Music in the second hymn is lines 15 to 17, and lines 23 ("Amphi") to 26 ("korai"). The tonic for both sections is on low g ("korai" ends on low g), and the "contrasting sound" is based on the note a .

¹⁹ West 1992a, 223-233 wrongly separates the Perfect System from the *tonoi*. They should go together.

improperly presented as enharmonic scales. The following is my best attempt to explain how this could have happened.

My Two-Source Hypothesis

Aristides' Damonian scales come down to us in two forms: description and notation. The description is a series of interval sizes. For example, the description of Iastian (*e***f**-a-c-d*) starting from the bottom, in parts of a tone, is: $\frac{1}{4}$, $\frac{1}{4}$, 2, $1\frac{1}{2}$, 1. The scales are also recorded in notation. But the notation is flexible and does not distinguish between enharmonic and chromatic. My Two-Source Hypothesis requires that the scales be recorded first in notation, which allows for a chromatic Lydian and Tense Lydian. Then later, a second source interprets all of the notated scales as enharmonic, mistakenly including Lydian and Tense Lydian, putting us in the Cave. Now despite the order of presentation that Aristides gives us (first description, then notation), Winnington-Ingram says that it is highly probable that Aristides found both description and notation in his source. But if he only found one of the two, he more likely would have found the notation, not the description (Winnington-Ingram 1936, 27). This is consistent with my own hypothetical scenario: somebody (probably not Aristides) comes across the Damonian scales in notated form only, and then applies the enharmonic description to all of them.

We actually have two sources: one Damonian (my earlier source) and one Aristoxenian (my later source). Damonians are a separate school of *harmonikoi* (harmonic theorists) within the broader *harmonikoi* movement of the fourth century BCE (Wallace 2015, 97-100). Their concern is ethos: how music affects the soul. Aristides' Book Two is mostly Damonian material. Book One is mostly Aristoxenian material, but the

Damonian scales are inserted into chapter 9. This supports my position: the notated scales are interpreted by an Aristoxenian. These scales are connected to Damon in Book Two, chapter 14, which allows Winnington-Ingram and West to conclude that they probably come from Damon (Winnington-Ingram 1936, 59 with n. 2; West 1002, 174, 247 with n. 81). I am more inclined to think that the scales come from Damon's followers. There is no evidence that Damon published (Wallace 2015 183-185), and the scales were likely recorded after Plato's *Republic*, as they match Plato's modes in name and number. More specifically, I think they come after the notational *tonoi* (mid-fourth century) since most of the scales are notated in the Lydian key, which is not a pre-*tonoi* key.²⁰

The important thing to notice is that these two sources have different views of the genera. Aristoxenian *harmonikoi* seemingly always work in the enharmonic genus. Unfortunately, we do not have many sources of Damonians. Aristotle's *Politics* is one source (Barker 1989, 175-176). But the important source is the Hibeh Papyrus on music,²¹ a widely recognized document of Damonian views, in which people call themselves *harmonikoi* and who use both enharmonic and chromatic music. Apparently, these Damonians are not Cave dwellers. So, my two sources are: an earlier Damonian who uses a flexible notation capable of enharmonic or chromatic music, and a later Aristoxenian who uses an inflexible, enharmonic description.

²⁰ The original pre-*tonoi* signs were an inner core of instrumental Hypolydian signs. After the introduction of the *tonoi*, Lydian became a preferred key because it incorporated the original Hypolydian key as its hypo-key for modulation. Hypolydian lacked this option. A discussion of the notation is beyond the scope of this paper.

²¹ Barker 1094, 183-185; West 1002, 247-248; Wallace 2015, 97-100.

Here is my Two-Source Hypothesis: Sometime after the advent of the notational *tonoi* (mid-fourth century BCE), a Damonian records the modes of Plato's Republic in Alypian notation (the ancient musical notation), which allows for both an enharmonic or a chromatic interpretation. Later, these notated scales come into the hands of an Aristoxenian, who then mistakenly applies the enharmonic description to *all* of the scales, putting us in the Cave.

Why would someone apply the enharmonic description? Here are some possibilities.

- “The *harmonikoi* before Aristoxenus only used the enharmonic genus. Plato's Republic was before Aristoxenus. Therefore, the scales in the Republic must be enharmonic.” This is an Aristoxenian who struggles with logic.

- “Sheer carelessness.” If most of Aristides' scales are enharmonic, then maybe the two chromatic scales, Lydian and Tense Lydian, were also interpreted as enharmonic by mistake. I think this easily could have happened.

- “Aristoxenus emphasizes the enharmonic.” All of his extant propositions are in the enharmonic. He says it is the finest of the genera, and that nobody appreciates it. He complains that the *harmonikoi* only use the enharmonic, and jokes that they deserve their “enharmonic” nickname. A naive Aristoxenian, overexposed to this enharmonic rhetoric, may be more likely to interpret Aristides' scales as enharmonic.

- “The ignorant interpreter of the Alypian notation.” This is someone who is unaware of the notation's enharmonic/chromatic flexibility. The notation is biased towards the enharmonic by default, especially for sharp keys, so that anyone more familiar with sharp keys would be more vulnerable to an enharmonic interpretation.

Now, whatever the reason, whoever applied the enharmonic description to Lydian and Tense Lydian created the illusion of an Enharmonic Cave. Shame on them. But we have been guilty not only of living in that illusory Cave, but of furnishing it with all the comforts of home, making it cozy, thus magnifying our problems. Shame on us. Because we *love* living in the Enharmonic Cave, and I can think of no better illustration of slavish Cave-Love than our blind acceptance of the Westphal Emendation in regard to Eratocles.

The Westphal Emendation

Eratocles is associated with rotating intervals of enharmonic octaves, which generates seven arrangements (species) of the octave. He does this presumably to enumerate different types of scales. The following excerpt is Aristoxenus complaining about his predecessors. The pretext is describing the differences between scales:

No one else, up to now, has ever got to grips with this part of the science. *Eratocles did attempt an enumeration* in just one section of it, without demonstration; but we saw previously ... No one, as we said before, has dealt with any of these matters at all, *except that* Eratocles, taking one *systema* in one genus, *attempted to enumerate the arrangements of the octave*, displaying them by moving the intervals around cyclically, but without demonstration. *He did not understand that unless the arrangements of the fifth and the fourth have also been demonstrated*, along with the form of combination [synthesis] according to which they can be melodically put together, his account will generate many times the seven arrangements. (Barker 1989, 130-13, *italics mine*)

Aristoxenus singles out Eratocles twice for making the attempt to enumerate different types of scales, in this case, arrangements of the enharmonic octave.²² But he criticizes Eratocles for not enumerating and demonstrating the arrangements of the fifth and the fourth, and how they can be melodically put together. Eratocles also ignored the chromatic and diatonic genera. But at least Eratocles made the attempt, which is more than can be said for other people, because here is Aristoxenus again:

...as to the distinctions between *systemata*, some people did not even attempt to enumerate them, devoting their research only to the *heptachords* which they called *harmoniai*; whereas others made the attempt, but by no means achieved a complete enumeration, these being people like the followers of Pythagoras of Zacynthos and Agenor of Mytilene.²³ (*italics mine*)

To which group of theorists does Eratocles belong? The first group, who did not even attempt to enumerate scales, or the second group, who made the attempt but did not achieve a complete enumeration? Clearly, Eratocles belongs to the second group, because he made the attempt, but ignored the fifth, the fourth, the chromatic, and the diatonic. So Eratocles is a follower of Pythagoras and Agenor, is he not?

Now here comes the Rudolf Westphal Emendation of 1883. This is how far back our modal problems go, 1883, an astonishingly long time ago! Heptachords are seven strings or seven notes of the kithara or lyre. Westphal changes

²² The enharmonic can be inferred from Barker 1989, 126-127.

²³ Barker 1989, 153 without the Westphal Emendation in n. 29. *Harmoniai* are scales, an aspect of mode.

“heptachords” to “seven octochords.”²⁴ As a result, “seven” no longer refers to seven strings or seven notes, but to seven “octochords” (octave species). Westphal is moving Eratocles and his seven octochords from the second group of theorists up to the first group, where he does not belong. Eratocles clearly belongs to the second group, because he uses his seven octochords to make an incomplete enumeration of scales. Westphal puts Eratocles and his seven octochords in the first group because he (Westphal) thinks the seven octochords should have modal names, *harmoniai* (Dorian, Phrygian, Lydian, etc.). But why should he automatically assume that seven is the correct number for the modes (*harmoniai*)? Plato only has six modes. Three *tonoi* systems before Aristoxenus imply only five or six modes. For whatever reason, Eratocles enumerates seven enharmonic octaves, but he does not give them modal names. Westphal gives them modal names, hence they are Westphal’s modes, not Eratocles’ modes. Carelessly assigning modal names to enharmonic octave species creates problems (see figure 4).

²⁴ Barker 1989, 153 with n. 29; cf. Barker 2007, 43, n.13. Westphal 1883, 448 apparently working with a different manuscript, changes “seven heptachords” to “seven octochords.” But Barker confirms that the original manuscript just had “heptachords”.

Assigning Modal Names to Octave Species

Enharmonic					Diatonic							
B*C	E*F	A	B	Mixolydian	B	C	D	E	F	G	A	B
*C	E*F	A	B*	Lydian	C	D	E	F	G	A	B	C
C	E*F	A	B*C	Phrygian	D	E	F	G	A	B	C	D
E*F	A	B*C	E	Dorian	E	F	G	A	B	C	D	E
*F	A	B*C	E*	Hypolydian	F	G	A	B	C	D	E	F
F	A	B*C	E*F	Hypophrygian	G	A	B	C	D	E	F	G
A	B*C	E*F	A	Hypodorian	A	B	C	D	E	F	G	A

fig. 4 – Assigning Modal Names to Octave Species

Assigning modal names to diatonic octave species is fine (fig. 4, right side), because the tone of disjunction (A-B, in blue) has the correct position in each octave species. From Hypodorian, it moves up a whole tone to Hypophrygian, up a whole tone to Hypolydian, up a semitone to Dorian, up a whole tone to Phrygian, up a whole tone to Lydian, and up a semitone to Mixolydian. The seven octave species must have the tone of disjunction in these locations, because the octave species govern the *tonoi*. The *tonoi* derive their names and their spacing from the octave species. But assigning modal names to enharmonic octave species is not valid (fig. 4, left side). From Hypodorian, the tone of disjunction moves up a ditone, up a quartertone, up a quartertone, up a ditone, up a quartertone, and up a quartertone. In Lydian and Hypolydian, the tone of disjunction is a quartertone too high. Worse, in Phrygian and Hypophrygian, the tone of disjunction is a whole tone too high. For example, the diatonic Phrygian octave goes from *d* to *d* (right side of diagram), but the enharmonic Phrygian octave goes from *c* to *c* (left side). Sing up and down diatonic Phrygian, *d-e-f-g-a-b-c-d*, and then sing enharmonic Phrygian (without the stars), *c-e-f-a-b-c*. Phrygian now has two separate personalities, one “minory”

and one “majory.” When we assign modal names to enharmonic octave species, we end up with a schizophrenic Phrygian.²⁵

So, the Westphal Emendation is untenable, because of how it dislocates the tone of disjunction, and because, as we said before, Eratocles does not belong to the first group of theorists, he belongs to the second group. The reason why we continue to accept the Westphal Emendation is because we want modal scales that predate the octave species. The octave species seem very suspicious to us because of their regularity: a uniform compass, no missing or extra notes. We rightly assume that early modal scales may have had some irregularities: spanning different lengths, having gaps or extra notes. Well, with the two false enharmonic scales (Lydian and Tense Lydian), Aristides’ scales look like they could be early forerunners of the enharmonic octave species. But this is an illusion (see figure 5).

The Imaginary Enharmonic Cave

Eratocles

B*C	E*F	A	B
*C	E*F	A	B*
C	E*F	A	B*C
E*F	A	B*C	E
*F	A	B*C	E*
F	A	B*C	E*F
A	B*C	E*F	A

Aristides

Mixolydian	B*C	D	E*F		B
Lydian			E*F	A	
Phrygian	D	E*F	A	B*C	D
Dorian	D	E*F	A	B*C	E
Hypolydian		*F	A	B*C	E*
Hypophrygian					
Hypodorian					

The Westphal Emendation

fig. 5 – The Imaginary Enharmonic Cave

The false enharmonic scales are in red. Iastian is not included because there is no octave species named Iastian. The Westphal

²⁵ And if we also carelessly assign modal names to chromatic octave species, then our schizophrenic Phrygian adds a third “bluesy” personality: *c[#]-e-f[#]-a-b-c-c[#]*. The same is true for the schizophrenic Hypophrygian.

Emendation assigns the modal names in the center to Eratocles' scales on the left. If we compare the right side to the left side, Aristides' red scale, $f-a-b^*c-e$, exactly matches Eratocles. Aristides' Dorian (cancel the d) can match Eratocles. Aristides' Mixolydian (cancel the d , add the a) can match Eratocles. Aristides' other red scale, e^*f-a-c , looks similar to Eratocles' scale. It is missing two lower notes ($*c$), and its highest note (c) should be the star ($*$) a quartertone lower, to match Eratocles. Also, its notated key implies that bb , not b , should be between the notes a and c . So, we have to pretend that it should have been notated in a different key in order to obtain the b . But Aristides' Phrygian can never match Eratocles, because its octave should be $d-d$, not $c-c$. So with the two false red scales, four out of five scales of Aristides can more or less match those of Eratocles.

But by correcting the two false red scales, only Mixolydian and Dorian can match the scales of Eratocles. The red scale, $f-a-b^*c-e$, is what Aristides calls his "Lydian" scale, and it should be chromatic: $f-f^\sharp-a-b-c-c^\sharp-e-f$. The other red scale, e^*f-a-c , is what Aristides calls "Tense Lydian," and it should also be chromatic: $e-f-f^\sharp-a-c$.

Why is the Enharmonic Cave in figure 5 imaginary? Because the two red scales should be chromatic, and the Westphal Emendation (assigning modal names to Eratocles), as we have already seen, lacks justification. We started out on the right side with just two falsely described scales (a relatively small historical error, I emphasize), and it seemingly put us in an Enharmonic Cave. And then we ended up on the left side, with the Westphal Emendation, needlessly burying ourselves even further in the Cave, because now every enharmonic octave species must have a modal name. We can walk out of this

imaginary Enharmonic Cave at any time: dump Westphal's modal names and change the two red scales back to chromatic.

Barker's "Empiricist" Interpretation

I conclude this paper with a discussion of Andrew Barker, because he buried us the most in the Cave, which is hard for me to say. His writings introduced me to this material back in 2010, and we tend to follow the opinions of those who introduce us to something new. Barker does not share West's view that Aristides' scales come from Damon. He completely denies the Damonian source. For him, there is only one source: an Aristoxenian source, which transforms the Damonian scales into "empiricist" scales.

Aristoxenian *harmonikoi* are empiricists: they derive their musical scales from what they actually hear. According to Barker, sometime in the late fifth century BCE, one of these empiricists listened to different kinds of music of the time, and recorded the scales of this music onto quartertone-grid diagrams. Despite chromatic music for the *kithara* (Barker 1984, 225), and for the New Music, which was flourishing at this time, this empiricist only heard enharmonic music. Moreover, these "empiricist" scales exactly matched the modal names of Plato's *Republic*, yet the *Republic* would not be written for another 40-50 years. Lastly, these diagrams of modal scales somehow survived for the next one hundred years, until the late fourth century, where they landed in the lap of Aristoxenus, who then applied the description of the scales, but not the notation, since he seemed to dismiss the notation as trivial. Somebody else notated the scales later (Barker 2007, 45-48, 51, and 1989, 155-157).

I say no. Aristides' scales were recorded after the *Republic*, since they exactly match Plato's modes, at a time when chromatic music was still more likely to be heard, allowing the possibility that some of the scales might be chromatic (Lydian and Tense Lydian). Also, Barker's empiricist scales imply that Aristides had two sets of scales, one Damonian set, which he did not give us, and one empiricist set. Because when Aristides says in Book Two, "Certainly, in the *harmoniai* which he [Damon] handed down, we can see that ...," (Barker 1989, 483) he is referring to scales that he did not bother to give us, which is very unlikely. West argues against Barker on this notion of two sets of scales (West 1992b, 46). Finally, Barker's empiricist scales imply that Plato did not get his modes from Damon, even though he got his rhythms from Damon. The consensus view is that Plato got both his modes and his rhythms from Damon. In fact, Barker admits that his non-Damonian modal interpretation is unorthodox (Barker 2007, 47 with n. 18).

Plato's Cave was the perfect metaphor for this paper. Some scholars who are heavily invested in the Cave will continue to argue for why we should stay in it and have our modal aspirations squashed. My hope is that this paper appeals to those who are not afraid to walk out. I believe I have shown a way. Try it. Our modal dreams may come true. Why stay in the Cave when as little as two incorrectly described scales stand in the way? Has two thousand years not been long enough? Is anyone craving another two thousand years? Come join me in the sunshine and let us give this Greek tragedy a happy ending.

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